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F-313

M.A./M.Sc. (First Semester)
EXAMINATION, Dec. - Jan., 2021-22
MATHEMATICS

Paper Fifth

(Advanced Discrete Mathematics - I)

Time : Three Hours] ***[Maximum Marks:80***

[Minimum Pass marks :16

Note : Attempt all sections as directed.

Section - A

(Objective/Multiple Choice Questions)

(1 mark each)

Note: Attempt all questions.

Choose the correct answer:

1. The preposition $\sim(P \wedge (\sim P))$ is a: -
   (A) Tautology
   (B) Contradiction
   (C) Both (A) and (B)
   (D) None of these

2. Consider the statement $p \to q,$, then $\sim q \to \sim p$ condition is called:
   (A) Direct implication
   (B) Inverse implication
   (C) Contra positive implication
   (D) Converse implication
3. Let g be a homomorphism from $(X, \bullet)$ to $(Y, *)$., If $g: X \to Y$ is one to one, then g is called:
   (A) Epimorphism
   (B) Monomorphism
   (C) Isomorphism
   (D) Endomorphism
4. The negate of the statement $(\forall x(x \neq 1, x \neq 2))$ is:
   (A) $\forall x(x^2 - 3x + 2 = 0)$
   (B) $\forall x(x = 1,\ x = 2)$
   (C) $\exists\, x(x = 1, x = 2)$
   (D) $\exists\, x(x^2 - 3x + 2 = 0)$
5. Which one is not correct?
   (A) The algebric system $< S, \bullet >$ is called a semigroup, if the operation '$\bullet$' is associative.
   (B) An element 'x' in a semigroup $< S, * >$ idempotent if $x^2 = x$.
   (C) Let $< S, * >$ and $< T, \Delta >$ be any two semigroups. A map $f : S \to T$ s.t. for $a, b \in S,\ f(a * b) = f(a) \Delta f(b)$ is called a semigroup homomorphism.
   (D) A semigroup $< M, \bullet >$ with an inverse element w.r.t. the operation '$\bullet$' is called monoid.

6. For any commutative monoid $< M, * >$, the set of idempotent elements of M form a_____

   (A) Semi monoid

   (B) Sub monoid

   (C) Sub semigroup

   (D) Semi group

7. The direct product of any two semigroup is a:

   (A) Semi group

   (B) Sub group

   (C) Sub monoid

   (D) Semi sub group

8. Let $< M, *, e >$, and $< T, \Delta, e' >$ be any two monoids, then a mapping f: $M \to T$, for $a, b \in M$, such that - $f(a * b) = f(a) \Delta f(b)$ and $f(e) = e'$ is called:

   (A) Monoid Homomorphism

   (B) Group Homomorphism

   (C) Semigroup automorphism

   (D) Semi group homomorphism

9. The Join irreducible element of a lattice L_1 which immediately succeed 'O' are called

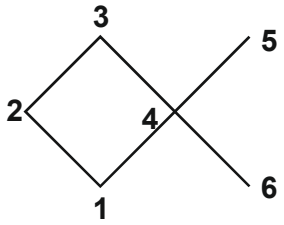
   (A) Cover of an element

   (B) Atom

   (C) Meet irreducible

   (D) None of these

10. The maximal and minimal elements of poset are -



   (A) 3,5 are maximal and 2,6 are minimal

   (B) 3,2 are maximal and 1,6 are minimal

   (C) 3,5 are maximal and 1,6 are minimal

   (D) 3,4 are maximal and 1,6 are minimal

11. Choose the incorrect statement -

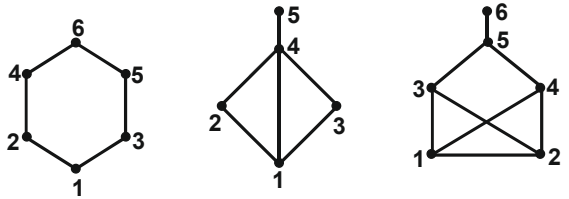
   (A) Every chain is distributive lattice

   (B) Every finite lattice is incomplete

   (C) Two bounded lattices L_1 and L_2 are complemented iff $L_1 \times L_2$ is complemented

   (D) Every well ordered set is totaly and complete order.

12. Which of the poset show in the figure below are lattices



(i) (ii) (iii)

   (A) (i) (B) (ii)

   (C) (iii) (D) (i), (ii), (iii) all

13. The conjuctive normal form of the function

$(x+y')(x'+y')(x'+y)$ is:

(A) $x'y'$ (B) $(x'+y)$

(C) xy' (D) $(x+y')$

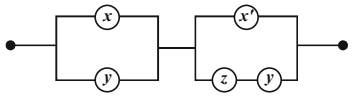
14. Let $(L_1 \leq)$ be a lattice, if for all a, b, c in L,

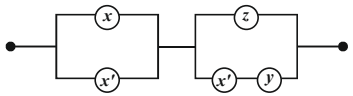
$a \leq c \Rightarrow a \vee (b \wedge c) = (a \vee b) \wedge c$. This type of lattice is said to be:

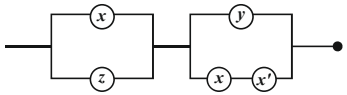
(A) Bounded lattice (B) Complete lattice

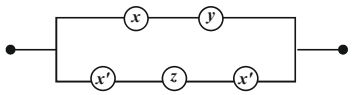
(C) Distributive lattice (D) Modular lattice

15. Which one is correct design of circuit for $(x+y)(x'+zy')$

(A) 

(B) 

(C) 

(D) 

16. The value of $\{a.b + a.b' + a'.b + a'.b'\}$ in boolean algebra $(B, +, \bullet)$ is

(A) $a'.b$

(B) $(a+b')$

(C) 1

(D) 0

17. If $L(G) = \{a^n b^n : n \geq 1\}$; then

(A) L (G) is regular

(B) L (G) is not regular

(C) L (G) is reduced grammar

(D) L (G) is non redundant

18. Grammar of Type - 1 is often called:

(A) Context free grammar

(B) Regular grammar

(C) Context sensitive grammar

(D) Regular Expression

19. The principle that shows whether a language is regular or non regular is:

(A) Kleen's theorem

(B) Lagrange's theorem

(C) Schwarz's Lemma

(D) Pumping Lemma

20. Which is not correct in regular expression-

(A) $R*R* = R$

(B) $RR* = R*R$

(C) $\phi R = R\phi = \phi$

(D) $\varepsilon + RR* = R* = \varepsilon + R*R$

Section - B

(Very short answer type questions)

(1½ marks each)

Note : Attempt all questions. Answer in 2 - 3 sentences.

1. Construct the truth table for $(\mathrm{P} \vee \mathrm{Q}) \vee \sim \mathrm{P}$.
2. Define predicates.
3. Define sub semigroup.
4. Explain congruence relation.
5. Draw Hasse diagrams of lattice with five elements.
6. Show that every finite lattice is bounded.
7. Design of circuit for $(xy + z)(x' + zy')$.
8. Write the function $(x + x'y)$ in conjuctive normal form.
9. Define regular grammar.
10. Write statement of Kneels Theorem.

Section - C

(Short Answer Type Questions)

(2 ½ marks each)

Note: Attempt all questions. Answer in less than 75 words.

1. Define Quantifiers. Explain types of quantifiers.
2. Show that the following argument is valid.

$$\begin{array}{l} p \vee q \\ \underline{\sim p} \\ q \end{array}$$

3. Show that the direct product of any two semigroups is a semigroup.
4. Prove that any commutative monoid $< M, * >$, the set of idempotent elements of M forms a submonoid.
5. Show that Every chain is distributive lattice.
6. Show that in Boolean algebra the complement of each element if exist is unique.
7. Find complete disjunctive normal form in three variables, and show that its value is 1.
8. Draw a circuit for the following Boolean function and replace it by a simpler one:

$$F(x, y, z) = x.z + [y.(y' + z).(x' + x.z')]$$

9. Explain polish notations.
10. Explain Regular set.

Section - D

(Long Answer Type Questions)

(4 marks each)

Note: Attempt all questions. Answer using less than 150 words for each.

1. Explain logical equivalence. Show that the statement is logically equivalent. $p \Rightarrow (q \Rightarrow r) \equiv (p \wedge q) \Rightarrow r$

OR

Test the validity of the argument: If 8 is even then 2 does not divide 9. Either 7 is not prime or 2 divides 9. But 7 is prime, therefore, 8 is odd.

2. Prove that $<M,*,e>$ and $<T,\Delta,e'>$ be two monoids with identities e and e' if f is an onto mapping from M to T i.e. $f : M \to T$ is an isomorphism. Then $f(e) = e'$

OR

Show that Let $<M,*>$ be monoid then there exists a subset $T \subset M^m$, such that $<M,*>$ is isomorphism to the monoid $<T,\bullet>$

3. Define the terms and give examples:

   (i) Join irreducible element of a lattice.

   (ii) Complemented lattices.

OR

Show that two bounded lattices L_1 and L_2 are complemented iff $L_1 \times L_2$ is complemented.

4. What is minimization of Boolean function? Explain Karnaugh method to represent Boolean function in one, two and three variable.

OR

Simplify the following Boolean expression

$$E(x_1, x_2, x_3) = x_1' x_2' x_3' + x_1' x_2 x_3' + x_1 x_2 x_3' + x_1 x_2' x_3.$$

5. Explain Grammar and their types.

OR

State and prove pumping lemma.